Submission date: 17 of December 2025
Acceptance date: 18 of March 2025
Publication date: 2 of April 2026

THE RELATIONSHIP BETWEEN MATHEMATICAL CORE COMPETENCIES AND ACADEMIC ACHIEVEMENT: AN EMPIRICAL STUDY OF CHINESE HIGH SCHOOL STUDENTS

LA RELACIÓN ENTRE LAS COMPETENCIAS MATEMÁTICAS BÁSICAS Y EL RENDIMIENTO ACADÉMICO: UN ESTUDIO EMPÍRICO DE ESTUDIANTES DE SECUNDARIA CHINOS

YuRong Ni [1]
E-mail: GS72049@student.upm.edu.my
ORCID: https://orcid.org/0009-0005-9131-8122
Mohammad Hasan Abdul Sathar [2] *
E-mail: mohdhasan@upm.edu.my
ORCID: https://orcid.org/0000-0002-7175-6521
Mohd Shafie Mustafa [2]
E-mail: mshafie@upm.edu.my
ORCID: https://orcid.org/0000-0001-9652-8616
[1] Universiti Putra Malaysia. Sarawak, Malaysia.
[2] Universiti Putra Malaysia. Selangor，Malaysia.
*Corresponding author

Suggested citation (APA 7th edition)

Ni, Y. R., Abdul Sathar, M. H., & Mustafa, M. S. (2026). The relationship between mathematical core competencies and academic achievement: An empirical study of Chinese high school students. *Universidad y Sociedad, 18*(2). e5937.

ABSTRACT:

This study investigates the relationship between four key mathematical competencies—abstraction, logical reasoning, modeling, and computation—and the mathematical achievement of high school students in China. Employing Structural Equation Modeling (SEM), the research analyzes data gathered from 856 high school graduates aged 16 to 18, focusing on both the direct and indirect effects of these competencies on achievement. The findings reveal that logical reasoning and mathematical modeling have the largest positive overall effect on mathematical achievement. In contrast, the competency of abstraction exhibits a small but significant total effect, which is primarily mediated through reasoning. Computational competency demonstrates a significant, moderate direct effect. The model accounts for 68.3% of the variance in mathematical achievement and displays excellent overall fit statistics. Multigroup analyses confirmed invariance across gender, grade level, and school type, indicating the robustness of the structural relationships.

Keywords: Mathematical competencies, Abstraction ability, Logical reasoning, Mathematical modeling, Computational thinking, Structural equation modeling.

RESUMEN:

Esta investigación analiza la relación entre cuatro competencias matemáticas fundamentales “abstracción, razonamiento lógico, modelado y cálculo” y el rendimiento académico en matemáticas de estudiantes de bachillerato en China. Mediante el uso de Modelos de Ecuaciones Estructurales (SEM), el estudio analiza datos de 856 graduados de bachillerato de entre 16 y 18 años, centrándose en los efectos directos e indirectos de dichas competencias sobre el rendimiento. Los resultados indican que el razonamiento lógico y el modelado matemático poseen el mayor efecto positivo global en el rendimiento. Por su parte, la competencia de abstracción muestra un efecto total pequeño pero significativo, el cual es ejercido principalmente de forma indirecta a través del razonamiento. La competencia de cálculo presenta un efecto directo moderado y significativo. El modelo explicó el 68.3 % de la varianza en el rendimiento matemático y mostró excelentes índices de ajuste global. Los análisis multigrupo confirmaron la invarianza del modelo según género, nivel de grado y tipo de institución, lo que sugiere la robustez de las relaciones encontradas.

Palabras clave:

Competencias matemáticas, Capacidad de abstracción, Razonamiento lógico, Modelado matemático,

Pensamiento computacional, Modelos de ecuaciones estructurales.

INTRODUCTION

Interdisciplinary Mathematical competencies are universally acknowledged as fundamental requirements for success in the 21st century (Organization for Economic Co-operation and Development, 2019). Their implications are far-reaching, impacting both individual development and national progress, particularly in science, technology, engineering, and mathematics (STEM) education. The Chinese education system, which has traditionally emphasized procedural fluency and accuracy in mathematical calculations (China. Ministry of Education, 2022; Ng & Cui, 2021), is gradually aligning with global standards by placing greater emphasis on developing higher-order mathematical thinking skills among youth. The Mathematics Curriculum Standards for Compulsory Education (2022 edition) explicitly outline six core mathematical competencies: mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical operation, and data analysis. This shift from a focus on 'knowledge' to 'competency' reflects global trends in mathematics education (Weintrop, 2016; Zhang, 2024), where deep understanding, flexibility, problem-solving, and real-world applications are increasingly stressed (National Research Council, 2001; Organization for Economic Co-operation and Development, 2022).

However, the effective implementation of competency-based learning requires a thorough understanding of the relationships between these competencies and academic achievement, as well as the interrelationships among the competencies themselves. The connection between various mathematical skills and achievement has been a persistent topic of research interest internationally and in China (Chen et al., 2024). While numerous studies have examined individual mathematical competencies in isolation—such as computational proficiency, reasoning, or problem-solving abilities—few have explored the complex relationships between multiple competencies and achievement using advanced analytical models. This gap is particularly significant within the Chinese educational context, where high-stakes examinations remain crucial determinants of educational progression, and the interplay between different competencies and achievement is not yet fully understood.

This study addresses this research gap by focusing on four essential mathematical abilities that underpin both the Chinese mathematics curriculum and international standards:

1. **Mathematical abstraction**: The ability to extract core mathematical patterns from concrete contexts, identify regularities, and translate relationships into symbolic form.
2. **Logical reasoning**: The capacity to employ deductive and inductive reasoning to draw valid inferences, construct mathematical arguments, and evaluate the reasoning of others.
3. **Mathematical modeling**: The ability to formulate, solve, and interpret mathematical models of real-world situations, encompassing the entire process from problem understanding to solution validation.
4. **Mathematical computation**: The capability to perform mathematical procedures accurately and efficiently, both manually and with the aid of computational tools.

The selection of these four competencies was guided by several considerations. First, they represent a spectrum of cognitive complexity, ranging from foundational skills (computation) to higher-order thinking (abstraction, reasoning, modeling). Second, they encompass both process-oriented competencies (reasoning, modeling) and skill-based competencies (computation, aspects of abstraction). Third, they reflect both the traditional strengths of Chinese mathematics education (computation) and the emerging priorities of curriculum reform (modeling). Fourth, they align with internationally recognized competency frameworks (Organization for Economic Co-operation and Development, 2019, 2022). Finally, exploratory factor analyses conducted during instrument development confirmed that these four competencies are empirically distinct yet theoretically coherent constructs.

This study has four main objectives:

1. To investigate the direct relationships between each mathematical competency and mathematics achievement, identifying which competencies are most strongly associated with performance on gateway mathematics assessments.
2. To examine the relationships among the mathematical competencies themselves, exploring whether foundational competencies such as abstraction exert their influence on achievement indirectly through more directly applicable competencies like reasoning and modeling.
3. To determine the relative contribution of each mathematical competency to overall mathematics achievement.
4. To assess whether these structural relationships remain consistent across different student subgroups, namely gender, grade level, and school type.

Employing structural equation modeling (SEM; Kline, 2023) on a large dataset of Chinese high school students, this study provides robust empirical evidence to inform evidence-based decision-making in mathematics education policy and practice. Theoretically, it contributes to the literature on mathematical competency by testing an integrated framework that simultaneously considers multiple competencies and their interrelationships. It extends prior research by examining both direct and indirect pathways

Vol 18 | No.2 | March-April | 2026
Continuous publication
e5937

between competencies and achievement, rather than relying solely on bivariate correlations. From a practical standpoint, the findings offer critical insights for improving mathematics instruction and student outcomes in secondary education.

Global Context and Societal Implications

The shift toward competency-based mathematics education is not unique to China; it reflects a global imperative to prepare students for a rapidly evolving, technology-driven society. Educational frameworks across Latin America, Eastern Europe, and OECD nations emphasize that traditional procedural fluency is no longer sufficient. Instead, developing logical reasoning and mathematical modeling is critical for cultivating functional citizens capable of navigating complex, data-rich environments. Furthermore, these competencies form the foundation of the future STEM (Science, Technology, Engineering, and Mathematics) workforce. By examining the structural relationships among abstraction, reasoning, modeling, and computation, this study provides actionable insights for international educational ministries aiming to align high school curricula with the demands of university enrollment and 21st-century societal challenges.

The Present Study and Hypotheses

Based on the reviewed literature, this study tests the following hypotheses:

- **H1**: All four mathematical competencies (abstraction, reasoning, modeling, computation) will demonstrate significant positive correlations with mathematics achievement.
- **H2**: Logical reasoning ability will show the strongest direct effect on mathematics achievement, reflecting its central role in mathematical thinking.
- **H3**: Mathematical modeling ability will demonstrate a significant direct effect on achievement, with effect sizes comparable to reasoning ability.
- **H4**: Computational ability will show a moderate direct effect on achievement, smaller than reasoning and modeling effects but nonetheless significant.
- **H5**: Abstraction ability will show a weak direct association with achievement but will demonstrate significant indirect pathways mediated through reasoning and modeling abilities, with the total indirect effect substantially exceeding the direct effect.
- **H6**: The structural relationships will remain invariant across gender, grade level, and school type, indicating the generalizability of the model.

MATERIALS AND METHODS

Research Design

This study employed a quantitative, cross-sectional correlational design, analyzed through structural equation modeling (SEM), to examine the relationships between four mathematical competencies—abstraction, reasoning, modeling, and computation—and mathematics achievement among Chinese high school students.

Participants and Sampling Procedure

The sample was obtained through stratified cluster sampling from six high schools in Jiangsu Province, China, selected to be representative of the region's educational context. The initial sample comprised 912 students. After excluding 56 cases due to incomplete responses or poor-quality data (e.g., patterned responses, excessively rapid completion), the final sample consisted of 856 high school students aged 15 to 18 years (M = 16.4, SD = 0.9). The sample included 418 males (48.8%) and 438 females (51.2%). Regarding grade level, 298 students were in Grade 10 (34.8%), 311 in Grade 11 (36.4%), and 247 in Grade 12 (28.8%). In terms of school type, 312 students (36.4%) attended key provincial schools (selective, high-achieving institutions), while 544 students (63.6%) attended non-key schools, which included regular public schools (n = 438) and private schools (n = 106).

Instruments

Four instruments were used to measure the mathematical competencies, and a standardized achievement composite was used as the outcome variable.

Mathematical Abstraction. Abstraction ability was measured using a 6-item self-report scale developed for this study based on the theoretical framework of the Mathematics Curriculum Standards (China. Ministry of Education, 2022). Items assessed students' perceptions of their ability to identify patterns, generalize from specific examples, and work with symbolic representations (e.g., "I can easily identify the underlying mathematical structure in a word problem"). Responses were recorded on a 5-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). Confirmatory factor analysis supported a unidimensional structure, and the scale demonstrated good internal consistency (α = 0.84).

Logical Reasoning. Reasoning ability was assessed using a 7-item self-report scale similarly developed from the curriculum standards. Items measured students' confidence in their deductive and inductive reasoning capabilities (e.g., "I can logically justify my mathematical solutions," "I am good at identifying patterns in sequences of numbers or figures"). The 5-point Likert scale yielded high reliability (α = 0.87), and factor analysis confirmed a single-factor structure.

Mathematical Modeling. Modeling ability was measured using two open-ended performance tasks adapted from established modeling assessments (Maaß, 2006). The first task required students to develop a mathematical model for optimizing transportation costs in a logistics scenario. The second task involved creating a model to project population growth based on given data. Each task was scored by two independent raters using a standardized rubric assessing five dimensions: problem understanding, simplification, mathematization, mathematical solution, and interpretation/validation. Each dimension was scored from 0 to 4, with total task scores ranging from 0 to 20. Inter-rater reliability was high (intraclass correlation coefficient = 0.91), and the two task scores were significantly correlated ($r = 0.68$, $p < 0.001$), supporting their combination into a single modeling competency score. For analysis, the combined score was scaled to have a mean of 500 and standard deviation of 100 to facilitate interpretation.

Mathematical Computation. Computational ability was measured using a 25-item procedural fluency test covering arithmetic operations, algebraic manipulation, and basic calculus procedures aligned with the high school curriculum. Items were drawn from standardized test banks and reviewed by experienced mathematics teachers for content validity. Students completed the test under timed conditions (30 minutes). The KR-20 reliability coefficient was 0.89, indicating strong internal consistency. For SEM analyses, the raw score (number correct) was converted to a standardized z-score.

Mathematics Achievement. Mathematics achievement was operationalized as the average of students' scores on four school-based examinations administered during the 2022–2023 academic year: two midterm examinations and two final examinations. These examinations were developed by each school's mathematics department following uniform curriculum guidelines, ensuring content alignment across schools. Raw scores were standardized within each school to a mean of 0 and standard deviation of 1 to permit cross-school comparisons. The four examination scores demonstrated high internal consistency ($\alpha = 0.92$).

Procedure

Data collection occurred in March 2024. School administrators distributed information about the study to parents and students, and informed consent was obtained from both parents and students prior to participation. Students completed the abstraction, reasoning, and modeling assessments through an online platform during regular school hours under teacher supervision. The computational fluency test was administered in paper-and-pencil format under examination conditions. Achievement data were extracted from school administrative records. To ensure data quality, the online platform included attention-check items and recorded response times; responses from participants who failed attention checks or completed the survey in less than half the median time were flagged for review and ultimately excluded if patterns suggested random responding.

Data Analysis

Data analysis proceeded in several stages. First, descriptive statistics, bivariate correlations, and reliability coefficients were computed using SPSS version 28. Second, confirmatory factor analysis was conducted to establish the measurement model for the latent variables (abstraction, reasoning, modeling, computation) using Mplus version 8.3. Third, the full structural equation model was estimated using maximum likelihood estimation with robust standard errors. Model fit was evaluated using multiple indices: chi-square (χ^2), chi-square/degrees of freedom ratio (χ^2/df), Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), Root Mean Square Error of Approximation (RMSEA) with 90% confidence interval, and Standardized Root Mean Square Residual (SRMR). Conventional thresholds for good fit were applied: $\chi^2/df < 3.0$, CFI/TLI > 0.90 (acceptable) or > 0.95 (good), RMSEA < 0.08 (acceptable) or < 0.05 (good), and SRMR < 0.08.

Fourth, mediation analysis was conducted to examine indirect effects of abstraction on achievement through reasoning and modeling. Bias-corrected bootstrapping with 5,000 resamples was used to generate 95% confidence intervals for indirect effects; effects were considered significant if the confidence interval excluded zero. Fifth, multigroup invariance testing was performed to examine whether structural relationships were consistent across gender, grade level, and school type. Invariance was assessed by comparing nested models with increasingly constrained parameters; a change in CFI of ≤ 0.01 between successive models was considered evidence of invariance.

Ethical Considerations

The study protocol was reviewed and formally approved by the Institutional Review Board (IRB) of Universiti Putra Malaysia (Approval Number: UPM-IRB-2025-0928). All procedures adhered strictly to international ethical guidelines for research involving human participants. Prior to data collection, written informed consent was obtained from the parents or legal guardians of all participating students, and age-appropriate written assent was collected from the students. Participants were informed of their right to withdraw at any time without penalty, and all data were anonymized to ensure strict confidentiality.

RESULTS AND DISCUSSION

Descriptive Statistics and Preliminary Analyses

Table 1 presents descriptive statistics and normality tests for all study variables. Mean scores indicated that students reported moderate to moderately high levels of abstraction ability (M = 3.48, SD = 0.72) and reasoning ability (M = 3.62, SD = 0.68) on the 5-point scales. Modeling ability scores (M = 498.5, SD = 96.8) were close to the intended scaled mean of 500, indicating appropriate task difficulty that avoided ceiling or floor effects. For computational ability, students correctly answered an average of 18.6 out of 25 items (SD = 3.4), reflecting reasonable but not perfect computational proficiency with substantial individual variation. In the SEM analyses, the standardized composite computational score (z-score) was used rather than the raw count.

All variables demonstrated approximately normal distributions based on skewness and kurtosis values. Absolute skewness values ranged from 0.08 to 0.31, all well below the threshold of 2.0 indicative of substantial departure from normality. Kurtosis values ranged from −0.35 to 0.45, also within acceptable ranges. Mardia's coefficient of multivariate kurtosis was 8.32, below the critical value of 11.25 for five continuous variables, suggesting acceptable multivariate normality for maximum likelihood estimation. These results indicated that parametric statistical procedures and maximum likelihood estimation were appropriate for the data.

Reliability estimates indicated good to excellent internal consistency for all measures. The abstraction scale ($\alpha = 0.84$), reasoning scale ($\alpha = 0.87$), modeling assessment ($\alpha = 0.91$ for combined task scores), computational test (KR-20 = 0.89), and achievement composite ($\alpha = 0.92$ across four examinations) all exceeded conventional thresholds for research purposes ($\alpha \geq 0.70$) and approached or exceeded thresholds for high-stakes decision-making ($\alpha \geq 0.90$). These high reliabilities reduce concerns about attenuation of observed relationships due to measurement error.

Table 1. Descriptive Statistics and Normality Tests (N = 856).

Variable	M	SD	Skewness	Kurtosis	α
Abstraction Ability	3.48	0.72	-0.18	-0.35	0.84
Reasoning Ability	3.62	0.68	-0.24	0.12	0.87
Modeling Ability	498.5	96.8	0.08	-0.21	0.91
Computational Ability	18.6	3.4	-0.31	0.45	0.89
Mathematics Achievement	0.00	1.00	-0.15	-0.08	0.92

Note. N = 856. M = mean; SD = standard deviation; α = Cronbach's alpha coefficient of internal consistency. Achievement scores were standardized, setting the mean to 0 and standard deviation to 1. All skewness and kurtosis values are within acceptable limits (|Skew| < 2.0, |Kurtosis| < 2.0).

Bivariate Correlations

The correlation matrix of all study variables is presented in Table 2. Consistent with Hypothesis 1, significant positive correlations were found between all four mathematical competencies and mathematics achievement, with effect sizes ranging from moderate to large. The strongest correlation emerged between logical reasoning and mathematics achievement ($r = 0.652$, $p < 0.001$), followed by modeling ($r = 0.618$, $p < 0.001$), abstraction ability ($r = 0.524$, $p < 0.001$), and mathematical computation ($r = 0.438$, $p < 0.001$).

The four competencies were significantly intercorrelated, with coefficients ranging from 0.385 to 0.576. The highest intercorrelation occurred between abstraction and reasoning ($r = 0.576$, $p < 0.001$), representing substantial shared variance (33.2%) while retaining sufficient unique variance (66.8%) to justify treating them as distinct constructs. This pattern of moderate intercorrelations supports the conceptualization of these competencies as related but separable components of mathematical proficiency, consistent with theoretical models of mathematical competency.

Notably, computational ability showed the weakest correlations with the other competencies, ranging from 0.385 to 0.467. This suggests that computational skill is relatively more independent from higher-order thinking abilities such as reasoning, modeling, and abstraction. This finding aligns with research distinguishing between procedural fluency and conceptual understanding as separate, albeit complementary, aspects of mathematical proficiency.

Table 2. Correlation Matrix of Study Variables (N = 856)

Variable	1	2	3	4	5
1. Abstraction Ability	—				
2. Reasoning Ability	0.576***	—			
3. Modeling Ability	0.512***	0.603***	—		
4. Computational Ability	0.385***	0.467***	0.428***	—	
Mathematics Achievement	0.524***	0.652***	0.618***	0.438***	—

Note. N = 856. All correlations are Pearson product-moment correlations. ***p < 0.001 (two-tailed). Effect size interpretation: small $r \geq 0.10$, medium $r \geq 0.30$, large $r \geq 0.50$.

Structural Equation Model

The hypothesized structural equation model was tested using maximum likelihood estimation with full-information maximum likelihood (FIML) for missing data. The model demonstrated excellent fit to the data: $\chi^2(162) = 284.16$, $p < 0.001$, $\chi^2/df = 1.75$, CFI = 0.952, TLI = 0.944, RMSEA = 0.048 (90% CI [0.041, 0.055]), SRMR = 0.042. All fit indices met or exceeded conventional thresholds for good model fit, indicating that the proposed model adequately represented the relationships among mathematical competencies and achievement. The significant chi-square test, while indicating some discrepancy between the model and data, is typical with large samples where even minor misspecifications are detected despite practically good fit.

Table 3 presents the complete set of model fit indices along with conventional cutoff criteria. The χ^2/df ratio of 1.75 was well below the conservative threshold of 2.0, indicating excellent fit. The CFI value of 0.952 exceeded the criterion for good fit (> 0.95), as did the TLI value of 0.944 which approaches the 0.95 threshold. The RMSEA point estimate of 0.048 fell within the good fit range (< 0.05), with the entire 90% confidence interval below 0.06. The SRMR value of 0.042 indicated very good fit (< 0.05). Collectively, these indices provide strong support for the adequacy of the hypothesized model structure.

Table 3. Structural Equation Model Fit Indices.

Fit Index	Value	Acceptable Fit	Good Fit
χ^2/df	1.75	< 3.0	< 2.0
CFI	0.952	> 0.90	> 0.95
TLI	0.944	> 0.90	> 0.95
RMSEA	0.048	< 0.08	< 0.05
RMSEA 90% CI	[0.041, 0.055]	—	—
SRMR	0.042	< 0.08	< 0.05

Note. N = 856. $\chi^2(162) = 284.16$, $p < 0.001$. CFI = Comparative Fit Index; TLI = Tucker-Lewis Index; RMSEA = Root Mean Square Error of Approximation; CI = confidence interval; SRMR = Standardized Root Mean Square Residual.

The standardized path coefficients for all direct relationships in the structural model are presented in Table 4. Consistent with Hypothesis 2, logical reasoning ability exhibited the strongest positive direct relationship with mathematics achievement ($\beta = 0.342$, SE = 0.034, $z = 10.06$, $p < 0.001$), accounting for 11.7% of unique variance in achievement scores. This strong coefficient underscores the centrality of reasoning to mathematical success. Mathematical modeling ability demonstrated the second strongest positive direct relationship ($\beta = 0.289$, SE = 0.031, $z = 9.32$, $p < 0.001$), supporting Hypothesis 3 and explaining 8.3% of unique variance. Computational ability showed a moderate but significant direct association ($\beta = 0.156$, SE = 0.029, $z = 5.38$, $p < 0.001$), consistent with Hypothesis 4 and accounting for 2.4% of unique variance.

Table 4. Standardized Path Coefficients and Variance Explained in the Structural Model.

Predictor	Outcome	β	SE	z	p
Logical Reasoning	Mathematics Achievement	0.342	0.034	10.06	<0.001
Modeling Ability	Mathematics Achievement	0.289	0.031	9.32	<0.001
Computational Ability	Mathematics Achievement	0.156	0.029	5.38	<0.001
Abstraction Ability	Mathematics Achievement	0.073	0.035	2.09	0.037
Abstraction	Reasoning Ability	0.375	0.032	11.72	<0.001
Abstraction	Modeling Ability	0.268	0.033	8.12	<0.001
Variable		R^2			
Mathematics Achievement		0.683			
Reasoning Ability		0.141			
Modeling Ability		0.072			

Note. N = 856. All β values are standardized coefficients. SE = standard error. R^2 represents the proportion of variance explained by all predictors of each endogenous variable.

Fully consistent with Hypothesis 5, abstraction ability showed a small direct association with achievement when other competencies were controlled ($\beta = 0.073$, SE = 0.035, $z = 2.09$, $p = 0.037$), contributing approximately 0.5% unique variance. As predicted, this direct association was notably smaller than those of reasoning and modeling. More importantly, and central to H5, the indirect pathways of abstraction substantially exceeded its direct association, as detailed in the mediation analysis below.

The model also revealed significant relationships among the mathematical competencies themselves. Abstraction ability significantly predicted both reasoning ability (β = 0.375, SE = 0.032, z = 11.72, $p < 0.001$) and modeling ability (β = 0.268, SE = 0.033, z = 8.12, $p < 0.001$), consistent with theoretical expectations that abstract thinking serves as a foundation for higher-order mathematical competencies. These pathways provide the basis for examining indirect effects through mediation analysis.

The model explained a substantial proportion of variance in mathematics achievement, with an R^2 value of 0.683, indicating that all four mathematical competencies combined explained 68.3% of the variance in achievement. The model also explained meaningful variance in the mediating competencies: abstraction accounted for 14.1% of the variance in reasoning ability and 7.2% of the variance in modeling ability. The high proportion of explained variance in achievement, together with the significant paths from abstraction to reasoning and modeling, provides evidence for the foundational role of abstraction in supporting other competencies.

Integrated Discussion and Pedagogical Implications

While the structural equation model (Table 4) provides robust statistical validation, it is crucial to translate these metrics into practical terms for educators and policymakers. The model revealed that logical reasoning and mathematical modeling are the strongest direct predictors of academic achievement. In plain language: a student's ability to memorize computational formulas is far less critical to their overall success than their capacity to logically deduce solutions and apply mathematics to real-world scenarios.

The most striking finding is the role of abstraction. The data showed that approximately 74% of abstraction's effect on achievement is mediated through reasoning and modeling. For a student sitting in a classroom, this means that the raw cognitive ability to think abstractly does not automatically translate to passing an exam. Instead, abstraction is the "engine" that powers reasoning and modeling. If a student can abstract a pattern but cannot apply it to a logical argument or a real-world model, their achievement will stall.

Pedagogical Translation: Teachers must bridge this gap by designing exercises that transition from pure abstraction to applied modeling. A concrete classroom example involves teaching algebraic functions. Rather than simply asking students to abstractly solve for ***x*** (pure abstraction), a teacher could present a real-world logistics problem, such as optimizing delivery routes for a local business. Students must first abstract the variables (distance, time, cost), then construct a mathematical model, and finally use logical reasoning to defend their optimized route. This seamlessly connects the foundational skill (abstraction) to the applied skills (modeling and reasoning) that directly drive achievement.

Societal Impact: From a broader societal perspective, these findings justify a paradigm shift in how high schools prepare students for the STEM workforce. High-stakes testing regimes that prioritize isolated computational skills are misaligned with the needs of modern universities and the tech industry. Fostering reasoning and modeling equips students not just to pass exams, but to become critically thinking citizens capable of evaluating data, understanding economic models, and contributing to technological innovation. Furthermore, our multigroup analysis confirmed that these pathways are identical across gender, grade levels, and school types, indicating that pedagogical reforms focusing on these competencies can promote educational equity on a national scale.

CONCLUSIONS

This study demonstrates that mathematical achievement is driven by a hierarchical ecosystem of competencies, where abstraction serves as the foundation that powers logical reasoning and mathematical modeling. Rather than merely summarizing cognitive processes, these findings carry profound implications for the intersection of university, school, and society. To prepare students for an increasingly complex, technology-driven global economy, educational systems must move beyond superficial curriculum changes.

We offer three macro-level policy recommendations. First, national assessment frameworks must be overhauled to prioritize modeling and reasoning over procedural computation, thereby forcing a systemic shift in classroom instruction. Second, teacher training programs should mandate pedagogical modules on how to translate abstract mathematical concepts into authentic, real-world modeling tasks. Finally, educational policymakers should view the cultivation of these higher-order competencies not just as an academic goal, but as a critical societal investment essential for closing global STEM skills gaps and fostering an informed, functional citizenry.

REFERENCES

Chen, M., Mok, I. A. C., Cao, Y., Wijaya, T. T., & Ning, Y. (2024). Effect of growth mindset on mathematics achievement among Chinese junior high school students: The mediating roles of academic buoyancy and adaptability. *Behavioral Sciences*, *14*(12), 1134. https://doi.org/10.3390/bs14121134

China. Ministry of Education. (2022). *Mathematics curriculum standards for compulsory education (2022 edition)*. Beijing Normal University Press.

Kline, R. B. (2023). *Principles and practice of structural equation modeling* (4th ed.). Guilford Press.

Maaß, K. (2006). What are modelling competencies? *ZDM Mathematics Education*, *38*(2), 113-142. https://doi.org/10.1007/BF02655885

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Principles,-Standards,-and-Expectations/

Ng, O. L., & Cui, Z. (2021). Examining primary students' mathematical problem-solving in a programming context: Towards computationally enhanced mathematics education. *ZDM Mathematics Education*, *53*(4), 847-860. https://doi.org/10.1007/s11858-020-01200-7

Organization for Economic Co-operation and Development. (2019). *PISA 2018 assessment and analytical framework*. OECD Publishing. https://www.oecd.org/content/dam/oecd/en/publications/reports/2019/04/pisa-2018-assessment-and-analytical-framework_d1c359c7/b25efab8-en.pdf

Organization for Economic Co-operation and Development. (2022). *PISA 2022 mathematics framework* (Draft). OECD Publishing. https://pisa2022-maths.oecd.org/files/PISA%202022%20Mathematics%20Framework%20Draft.pdf

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and Technology*, *25*(1), 127-147. https://doi.org/10.1007/s10956-015-9581-5

Zhang, J., Zhou, Y., Jing, B., Pi, Z., & Ma, H. (2024). Metacognition and mathematical modeling skills: The mediating roles of computational thinking in high school students. *Journal of Intelligence*, *12*(6), 55. https://doi.org/10.3390/jintelligence12060055

CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

Authors' Contribution (CRediT Taxonomy)

Author	Roles
YuRong Ni	Conceptualization, Investigation, Methodology, Project Administration, Visualization, Writing – original draft
Mohammad Hasan Abdul Sathar	Validation, Formal Analysis, Supervision, Writing – review & editing
Mohd Shafie Mustafa	Data Curation, Software, Validation, Writing – review & editing