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METHODS

OF MODELING THE FLOW OF VISITORS AT PUBLIC EVENTS

MÉTODOS DE MODELIZACIÓN DEL FLUJO DE VISITANTES EN ACTOS PÚBLICOS

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ABSTRACT

This study aims to improve the visitor experience at large-scale mass events. The risks associated with emergencies at mass events can be significantly reduced with information-driven access control systems. In this context, a pressing research objective is to develop a mathematical model of visitor flow movement at the event area entrances and exits. The model for the passage of arriving visitors through the turnstiles is a multi-channel queuing system with unlimited queues. In this case, the flow of requests is not constant. A queuing system with varying inflow intensity can be modeled by approximating the inflow intensity using continuous piecewise functions. The paper describes numerical and numerical-analytical solution methods for the problem of finding state probabilities. The pedestrian movement model as they leave the event venue can be described by a multi-channel queuing system. The request flow and service time are distributed according to Erlang's law. The stationary probabilities of the system states are found by Markovization using the pseudo-state method. The authors present an algorithm for finding the stationary probabilities of the system using recurrent relationships and a method for calculating service organization quality characteristics for participants leaving the event area. The conclusions can be applied to the automated regulation of visitor flows using digital information panels.

Keywords: Pedestrian flow, Mass event, Mathematical model, Queuing system.

RESUMEN

El estudio pretende mejorar la experiencia de los visitantes en eventos multitudinarios a gran escala. Los riesgos asociados a las emergencias en eventos multitudinarios pueden reducirse notablemente con sistemas de control de acceso informativos. En este contexto, un objetivo apremiante de la investigación es desarrollar un modelo matemático del movimiento del flujo de visitantes en las entradas y salidas de la zona del evento. El modelo para el paso de los visitantes que llegan a través de los torniquetes es un sistema de colas multicanal con colas ilimitadas. En este caso, el flujo de solicitudes no es constante. Un sistema de colas con intensidad variable de flujos entrantes puede modelarse aproximando la intensidad del flujo entrante mediante funciones a trozos continuas. El documento describe los métodos de solución numérica y numérico-analítica al problema de encontrar las probabilidades de los estados. El modelo de movimiento de los peatones al abandonar el lugar del evento puede describirse mediante un sistema de colas multicanal. El flujo de solicitudes y el tiempo de servicio se distribuyen según la ley de Erlang. Las probabilidades estacionarias de los estados del sistema se hallan mediante markovización utilizando el método de los pseudoestados. Los autores presentan un algoritmo para hallar las probabilidades estacionarias del sistema utilizando relaciones recurrentes y un método para calcular las características de la calidad de la organización del servicio para los participantes que abandonan la zona del evento. Las conclusiones pueden aplicarse a la regulación automatizada de los flujos de visitantes mediante paneles informativos digitales.

Palabras clave: Flujo de peatones, Evento masivo, Modelo matemático, Sistema de colas.

INTRODUCTION

Large-scale cultural events are bound to increase the density of pedestrian and traffic flows in the event area. The ability to predict these changes makes it possible to better organize traffic for this period (Naumova, 2020), minimize the probability of traffic jams, and increase convenience for visitors to the event. An important role is played by modeling pedestrian flows in various situations arising during mass events. It is especially difficult to control the behavior of pedestrians in case of panic. To avoid such situations, event organizers need to carefully plan and study the peculiarities of visitors' behavior (Zhang & Jia, 2021) to predict the following parameters:

- evacuation time of spectators in different demarcation zones;
- the location, number, and characteristics of emergency exits;
- road capacity at the intersections of pedestrian and traffic flows.

The parameters that can be managed are service time, waiting time at visitor entry-exit points (queuing systems), the number of such systems on the route, and the filling rate of waiting areas.

The behavior of pedestrians has been studied using questionnaire surveys, empirical observations, control experiments, mathematical modeling, and simulation modeling (Duan, 2024). The latter two methods have the advantage of allowing to predict various standard and emergency situations, eliminating risks with minimal costs. The models of pedestrian flows created to date include microscopic, mesoscopic, and macroscopic models (Beritell et al., 2020; Naumova, 2020; Zhang & Jia, 2021). Each of these solves specific tasks and involves varying detalization of raw data.

Microscopic models require high detalization, as they account for the behavior of every individual and their interaction with others. Macroscopic models model pedestrian flow and deal with variables, such as speed, density, and intensity. Mesoscopic modeling combines microscopic and macroscopic aspects, examining each individual from the standpoint of the entire flow.

A relevant research objective is to develop a mathematical model of visitor flow for mass events that would predict the parameters of pedestrian flows and the quality of organization of their movement with sufficient accuracy.

The study's goal is to improve visitor experience at large-scale mass events.

The event location is divided into the venue and the event area. The venue can be, for example, a stadium or a concert hall. The event venue includes the zone of the event itself with its exits and channeled pedestrian flows. The event area or the last mile zone refers to adjacent territory and street and road networks within a radius of about 1,500 meters from the venue.

The risks associated with emergencies at mass events can be significantly lowered using informational access control systems (Zhang & Jia, 2021). The structure of such a system needs to satisfy the unique needs of the event and its location.

Informational access control systems allow managing visitor flows by, for example, providing directions for the movement of pedestrian flows on digital information boards. If the flow in one direction reaches a critical density, instructions for the possible directions of the flow can be changed.

The passage of the visitor flow through checkpoints can be modeled using queuing theory (Korelin et al., 2018). In terms of this classification, these models belong to the mesoscopic level. Queuing theory is used to investigate processes occurring in complex stochastic systems.

For safety purposes, before the start of a mass event, visitors pass through turnstiles. A common queue is formed at the entrance with n turnstiles. Thus, the model of passage through the turnstiles is a multi-channel queuing system with unlimited queuing. Service time can be considered to have an exponential distribution. However, the flow of requests cannot be considered stationary in this case. Research suggests that the intensity of the visitor flow increases monotonically from zero to some maximum. The maximum point is achieved 10–15 minutes before the start of the event. After this, it falls to zero.

Several studies (Duignan et al., 2023; Gnedenko & Kovalenko, 1966; Korelin & Porshnev, 2017, 2020; Stienmetz & Fesenmaier, 2018; Wang et al., 2024) propose modeling queuing systems (QS) with varying incoming flow intensity by approximating said intensity using piecewise-continuous functions. In this case, the work of a non-stationary QS can be described as the successive operation of stationary QSs, each of which activates the moment the other one is finished. The initial conditions, i.e., the probabilities of the system being in a certain state, will change.

The model of pedestrian movement when leaving the event venue and passing through narrow doorways (bottlenecks) can also be described using queuing systems. The density of pedestrian flow in this case is high and, according to studies, consistent with a normal distribution. The normal law is approximated by the Erlang law (Naumova & Saphonova, 2020) of at least the 5th order, which allows modeling queuing systems of varying complexity.

The most important characteristics that need to be determined are maximum queue length; maximum queue waiting time; time periods when the queue reaches maximum length and maximum waiting time; the number of requests serviced before the start of the mass event; and the time spent servicing all the received requests.

MATERIALS AND METHODS

In this study, the dynamics of pedestrian flow at large-scale mass events were analyzed using mathematical and simulation modeling. The research approach centered around queue theory and mesoscopic modeling methods, which strike a balance between individual behavior and overall crowd dynamics.

The movement of pedestrians at the point of entry was modeled using a multi-channel queue system with unlimited queue length. The incoming flow was characterized by non-stationary arrival models, which were approximated using piecewise-continuous functions to account for changes in intensity over time. Service time followed the Erlang law of distribution. The probabilistic states of the system were analyzed using markovization with pseudostate methods. This strategy made it possible to accurately predict flow patterns and the characteristics of the queue.

The study of exit points focused on high-density pedestrian flows passing through bottlenecks, such as narrow doorways. These flows were modeled as multi-channel queuing systems with the time of arrival and service time distributed according to Erlang law. State transitions were visualized with a state graph, and differential equations were made to describe the behavior of the system over time.

Differential equations were solved using the Euler and Runge–Kutta methods to improve accuracy.

Numerical and analytical approaches were utilized to derive stationary probabilities and to estimate system characteristics.

Indicators such as queue length, waiting time, and service speed were calculated to assess service quality and identify peak load periods.

RESULTS AND DISCUSSION

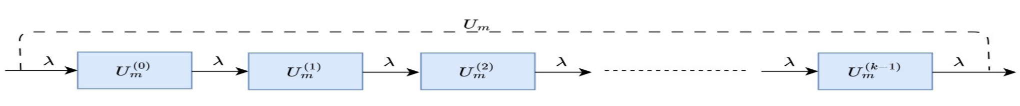
Modeling the movement of visitors through checkpoints

The model of visitors' passage through the turnstiles at the entrance to the event can be represented by a QS of the form $E_k/M/m/n$. Therefore, the incoming flow of requests (the flow of visitors approaching the entrance to the event) is a Palm flow with time intervals between consecutive events following a kth-order Erlang distribution. The service of requests (one visitor passing through a turnstile) has an exponential distribution. There is a total of m turnstiles (service equipment units). Queue length is limited (can include up to n = 100 - 150 people).

Let us draw up a system of differential equations for the states of the system. Let U_m denote the state of the system in which it has m requests. To markovize the process, we will use the method of pseudostates (Naumova & Saphonova, 2020; Prabhu & Zhu, 1989;). The incoming flow with a special Erlang distribution can be presented as a sum of k exponential distributions with λ as the parameter.

Pseudostates for U_m are shown in Figure 1.

Fig 1. Pseudostates of the kth-order Erlang distribution.



Source: own elaboration.

Let us make differential equations to find the probabilities of the system being in the states U_m (Naumova & Saphonova, 2020). The graph of states is given in Figure 2. Hereinafter, we will denote $p_s^{(0)} = p_s$ for all s.

1) U_{m+i} — all channels are occupied, i requests in the queue.

Let $P_{n+i}(t + \Delta t)$ denote the probability of the queue having i requests at the time point $(t + \Delta t)$ (F1).

$$P_{n+i}(t + Dt) » P(A) + P(B) + P(C), \quad \text{(F1)}$$

where:

A = {the system was in state ($m+i$) and nothing happened over time period Δt};

B = {the system was in state ($m+i-1$) and one new request was received over time period Δt};

C = {the system was in state ($m+i+1$) and one service channel was freed over the time period Δt}.

$$p_{m+i}(t+\Delta t) \approx p_{m+i}(t)\cdot(1-(\lambda+m\mu)\Delta t)+p^{(k)}{}_{m+i-1}(t)\cdot\lambda\cdot\Delta t+m\mu\cdot p_{m+i+1}(t)\Delta t \quad \text{(F2)}$$

Dividing (F2, F3) by Δt, we obtain:

$$\frac{p_{m+i}(t+\Delta t)-p_{m+i}(t)}{\Delta t} \approx -p_{m+i}(t)(\lambda+m\mu)+ p^{(k)}{}_{m+i-1}(t)\cdot\lambda+m\mu p_{m+i+1}(t). \quad \text{(F3)}$$

We obtain the system of differential equations (F4):

$$p'_{m+i}(t) = -p_{m+i}(t)\cdot(\lambda+m\mu)+p^{k}_{m+i-1}(t)\cdot\lambda+m\mu p_{m+i+1}, \ (i=1;2;3;...,n-1) \quad \text{(F4)}$$

2) Us — there is no queue and s (s x m) service channels are occupied

Ps(t + Δt) – the probability of the system being in this state. The system will be in state Us if the following events occur:

A = {over time period Δt, no requests were received and none of the m service channels were freed};

B = {one of the (s + 1)th occupied service channels was freed};

C = {(s – 1) service channels were occupied at time point t and one request arrived over time period Δt} (F5).

$$p_s(t+\Delta t) \approx p_s(t)\cdot(1-(\lambda+s\mu)\Delta t)+(s+1)\mu\, p_{s+1}(t)\cdot\Delta t+ p^{(k)}{}_{s-1}(t)\lambda\,\Delta t. \quad \text{(F5)}$$

From this, we obtain the differential equation (F6):

$$(p_s(t))'_t = -(\lambda+s\mu_0)p_s(t)+(s+1)\mu\cdot p_{s+1}+\lambda p^{(k)}_{s-1}(t) \qquad (s=1, 2, ..., m). \quad \text{(F6)}$$

3) U_0 — the system is completely free.

Similar to the previous case, we obtain the equation (F7):

$$(p_0(t))'_t = -\lambda\, p_0(t)+\mu p_1(t) \quad \text{(F7)}$$

4) U_{m+n} — all channels are occupied, there are n requests in the queue.

Pseudistate U_{m+n} is made up of a single subset $.U_{m+n} = \{U^{(0)}_{n+m}\}.$

For probability , we obtain the equation (F8):

$$p_{m+n}(t+\Delta t) \approx p_{m+n}(t)\cdot(1-m\mu\cdot\Delta t)+p^{(k)}{}_{m+n-1}(t)\cdot\lambda\cdot\Delta t \qquad \text{(F8)}$$

From this it follows that (F9):

$$(p_{m+n}(t))' = -m\mu\, p_{m+n}(t)+\lambda\, p^{(k)}_{m+n-1}(t) \qquad \text{(F9)}$$

5) being in pseudostate U_s

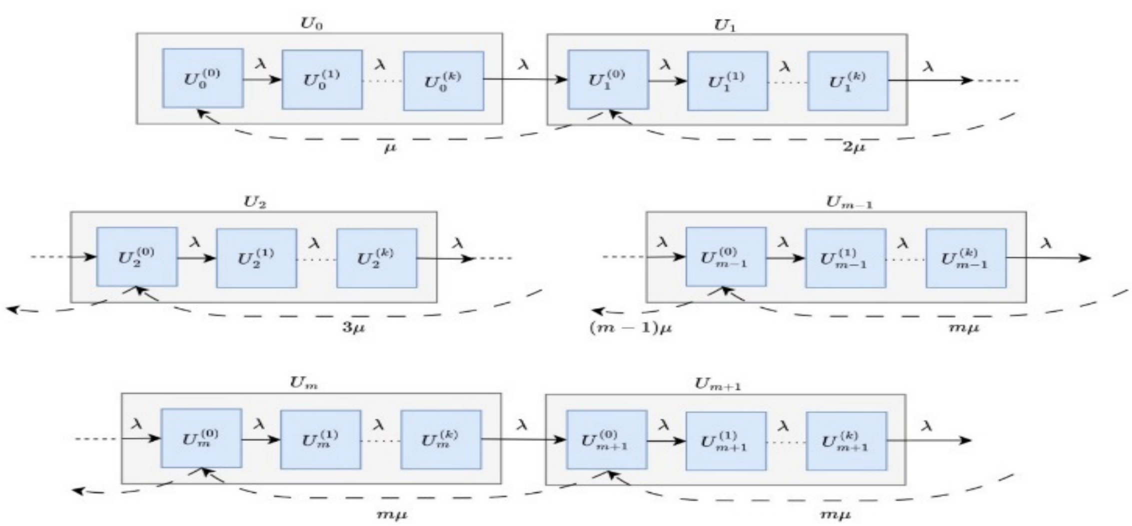
For the probabilities of being in transitional states , it is true that (F10):

$$p_s^{(j)}(t+\Delta t)\approx p_s^{(j)}(t)(1-\lambda\Delta t)+p_s^{(j-1)}(t)\cdot\lambda\,\Delta t \qquad \text{(F10)}$$

Dividing (Figure 2) by Δt, we find the limit at Δt (F11) :

$$\left(p_s^{(j)}(t)\right)'_t = -\lambda\, p_s^{(j)}(t)+\lambda\, p_s^{(j-1)}(t) \qquad (j=1,2,3,....,k) \qquad \text{(F11)}$$

Fig 2. Graph of QS states Ek/M/m/n.



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Thus, to determine the unknown probabilities, we obtain a system of differential equations (F12):

$$\begin{cases}(p_s(t))'_t = -(\lambda+s\mu)p_s(t)+(s+1))\mu\cdot p_{s+1}+\lambda\, p^{(k)}_{s-1},\ (s=1,2,...,m)\\(p_{m+i}(t))'_t = -(\lambda+m\mu)p_{m+i}(t)+\lambda p^{(k)}_{m+i-1}(t)+m\mu p_{m+i-1}(t),\ (i=1,2,3,...,n-1)\\(p_{m+n}(t))' = -m\mu\, p_{m+n}(t)+\lambda\, p^{(k)}_{m+n-1}(t)\\(p_s^{(j)}(t))'_t = -\lambda p_s^{(j)}(t)+\lambda p_s^{(j-1)}(t),\ (j=1,2,...,k)\end{cases} \qquad \text{(F12)}$$

model non-stationary visitor flow with the intensity of the flow varying over time, we will represent this flow as a piecewise-continuous function. To do so, we divide the time axis into intervals $[t_{(i-1)};\ t_i]$ and assume that the intensity of the requests inflow is constant within each interval. In this case, Erlang parameters for each of the intervals will also be stationary. For the ith interval, we will denote them by λ_i and k_i.

Accordingly, using the Heaviside step function, we can write down the dependence of distribution parameters on time as follows (F13, F14):

$$\lambda(t) = \sum_{i=0}^{K} (\theta(t - t_i) - \theta(t - t_{i+1}))\, \bar{\lambda}_i \quad \text{(F13)}$$

$$k(t) = \sum_{i=0}^{K} (\theta(t - t_i) - \theta(t - t_{i+1}))\, k_i \quad \text{(F14)}$$

where $\theta(t-a) = \begin{cases} 0, t < a, \\ 1, t \geq a \end{cases}$ Then, the performance quality characteristics of the QS are calculated using the formulas:

$L_q(t) = \sum_{n=m}^{\infty} (n - m) p_n(t)$ —the dependence of queue length on time;

$W_q(t) = \sum_{n=m}^{\infty} (n - m + 1) \frac{p_n(t)}{m\mu}$— the dependence of queue waiting time on time.

Thus, to solve the task at hand, we need to solve a system of Kolmogorov differential equations for a stationary QS under random initial conditions and write the p_n (t) probabilities using the Heaviside step function.

To determine the unknown probabilities of the QS at each of the intervals [$t_{(i-1)}$; t_i] , we need to solve a system of differential equations. The initial conditions are as follows (F15):

$$p_g^{(0)}(0) = 1,\ \ p_m^{(j)} = 0 \ \ (m = 0, 1, 2, .., g\text{-}1, g\text{+}1, , \ldots;\ j = 1,\ldots, k,\ \ j = 1, 2, 3, \ldots, q) \quad \text{(F15)}$$

The number g of probability , which is non-zero at the beginning of a new time interval [$t_{(i-1)}$; t_i] , is determined based on the precondition that g=[M(X(t))]— the integer part of the mathematical expectation of the number of requests X(t) in the system in the previous time interval [t_(i-2); t_(i-1)].

Here we should note that: $r_m(t) = p_m(t) + \sum_{j=1}^{k} p_m^{(j)}$

1) P_m^0=p_m in our notations, for all *m*;

2) $r_m(t)=P(U_m)$, meaning that $r_m(t)$ — the probability of the system being in state U_m;

3) according to probability theory: .

Numerical solution of the system of differential equations for state probabilities

There are different possible numerical solutions of the obtained system of differential equations (12).

Method 1. The system of linear differential equations (12) can be solved, for example, by the Euler method (F16).

$$f_s = -(\lambda + s\mu) p_s(t) + (s+1))\mu \cdot p_{s+1} + \lambda\, p_{s-1}^{(k)},\ (s = 1, 2, \ldots, m)$$

$$f_{m+i} = -(\lambda + m\mu) p_{m+i}(t) + \lambda p_{m+i-1}^{(k)}(t) + m\mu p_{m+i-1}(t),\ (i = 1,2,3,\ldots, n-1)$$

$$f_{m+n} = -m\mu\, p_{m+n}(t) + \lambda\, p_{m+n-1}^{(k)}(t) \quad \text{(F16)}$$

$$f^{(j)}{}_s = -\lambda p_s^{(j)}(t) + \lambda p_s^{(j-1)}(t),\ (j = 1,2,\ldots,k)$$

The calculation formulas will be as follows (F17):

$$\begin{cases} (p_s)_q = (p_s)_{q-1} + h \cdot f_s,\ (s = 1, 2, ..., m) \\ (p_{m+i})_q = (p_{m+i})_{q-1} + h \cdot f_{m+i},\ (i = 1,2,3,...,n-1) \\ (p_{m+n})_q = (p_{m+n})_{q-1} + h \cdot f_{m+n} \\ \left(p_s^{(j)}\right)_q = \left(p_s^{(j)}\right)_{q-1} + h \cdot f^{(j)}{}_s,\ (j = 1,2,...,k) \end{cases} \qquad \text{(F17)}$$

$p_g^{(0)}(0) = 1,\ p_s^{(j)} = 0$ (s = 0, 1, 2, .., g-1, g+1, , …; j = 1,…, k, $j = 1, 2, 3,...., q$

h— integration step, s — iteration number.

Alternatively, the system can be solved through the Runge–Kutta method, for example, of the 4th order (F18):

$$\begin{cases} (p_s)_q = (p_s)_{q-1} + \frac{1}{6}k_{s,1} + \frac{2}{6}k_{s,2} + \frac{2}{6}k_{s,3} + \frac{1}{6}k_{s,4},\ (s = 1, 2, ..., m) \\ (p_{m+i})_q = (p_{m+i})_{q-1} + \frac{1}{6}k_{m+i,1} + \frac{2}{6}k_{m+i.2} + \frac{2}{6}k_{m+i,3} + \frac{1}{6}k_{m+i,4},\ (i = 1,2,3,...,n-1) \\ (p_{m+n})_q = (p_{m+n})_{q-1} + \frac{1}{6}k_{m+n,1} + \frac{2}{6}k_{m+n,2} + \frac{2}{6}k_{m+n,3} + \frac{1}{6}k_{m+n,4} \\ \left(p_s^{(j)}\right)_q = \left(p_s^{(j)}\right)_{q-1} + \frac{1}{6}k^{(j)}{}_{s,1} + \frac{2}{6}k^{(j)}{}_{n+i.2} + \frac{2}{6}k^{(j)}{}_{s,3} + \frac{1}{6}k^{(j)}{}_{s,4},\ (j = 1,2,...,k) \end{cases} \qquad \text{(F18)}$$

Herein, the notations used for I=0,1,…,m+n are (F19):

$$k_{l,1} = h \cdot f_s\left(t_q, (p_l)_q\right), \quad k_{l,2} = h \cdot f_s\left(t_q + \frac{h}{2}, (p_l)_q + k_{l,1}\right),$$
$$k_{l,3} = h \cdot f_s\left(t_q + \frac{h}{2}, (p_l)_q + k_{l,2}\right), \quad k_{l,4} = h \cdot f_s\left(t_q + h, (p_l)_q + k_{l,3}\right), \qquad \text{(F19)}$$

And (F20):

$$k^{(j)}{}_{l,1} = h \cdot f_s\left(t_q, \left(p_l^{(j)}\right)_q\right), \quad k^{(j)}{}_{l,2} = h \cdot f_s\left(t_q + \frac{h}{2}, \left(p_l^{(j)}\right)_q + k^{(j)}{}_{l,1}\right),$$
$$k_{l,3} = h \cdot f_s\left(t_q + \frac{h}{2}, \left(p_l^{(j)}\right)_q + k^{(j)}{}_{l,2}\right), \quad k_{l,4} = h \cdot f_s\left(t_q + h, \left(p_l^{(j)}\right)_q + k^{(j)}{}_{l,3}\right), \qquad \text{(F20)}$$

This method requires four calculations of the right-hand side of the differential equation at each step. On the upside, since it is a 4th-order method, it reduces the computational error.

Method 2. Numerical and analytical method to solve the system of linear differential equations

The system of linear homogeneous differential equations in matrix form will look as follows (F21):

P(t)=A•P(t) (F21)

In this, matrix A is tridiagonal. The eigenvalues for a tridiagonal matrix can be found using the standard algorithm:

Step 1. Find the characteristic polynomial det(A- λ E)=0

For a tridiagonal matrix, there is a special way to compute the determinant det(A- λ E)=0 without explicitly expressing it as a polynomial.

Let $D_m(\lambda)$ be the principal minor of the ***m***th order of matrix $(A-\lambda E)$. Then:

$D_m(\lambda)=(a_{(m.m)}-\lambda)\,D_{(m-1)}(\lambda)-a_{(m.m-1)}\,M_{(m.m-1)}(\lambda)$— minor decomposition in the last line.

The additional minor $M_{(m.m-1)}(\lambda)$ for element $a_{(m.m-1)}$ in the last column contains one non-zero element $a_{(m-1.m)}$. Therefore, it can be decomposed in that column:

M_(m,m-1) (λ)=a_(m-1.m) D_(m-2) (λ) .

Thus, we obtain the recurrent formula to calculate the minors (F22):

$D_m(\lambda)=(a_{(m.m)}-\lambda)\,D(m-1)(\lambda)-a_{(m.m-1)}\,a_{(m-1,m)}\,D_{(m-2)}(\lambda)$, m=3,4,…,n, (F22)

Step 2. Find the roots of the characteristic polynomial; $D_n(\lambda)$

The roots of the polynomial can be found, for example, with the method of parabolas.

Step 3. Write out the solution of the system using the analytical approach for ordinary differential equation systems.

Importantly, in contrast to the numerical method, which gives a finite set of points, the solution of the system in this case involves the construction of a procedure that allows determining the probabilities of states at arbitrary moments of time.

Modeling pedestrian flow at the exit from the event area

Let us examine pedestrian flows at the exit from the event venue passing through narrow doorways (bottlenecks). This issue is especially relevant in case of emergency situations to organize evacuation in the optimal way (Zhang & Jia, 2021).

In this case, pedestrians move straight ahead in ***b*** rows: on one side and on the other. The flows merge in front of a doorway that can only fit ***m*** people at once.

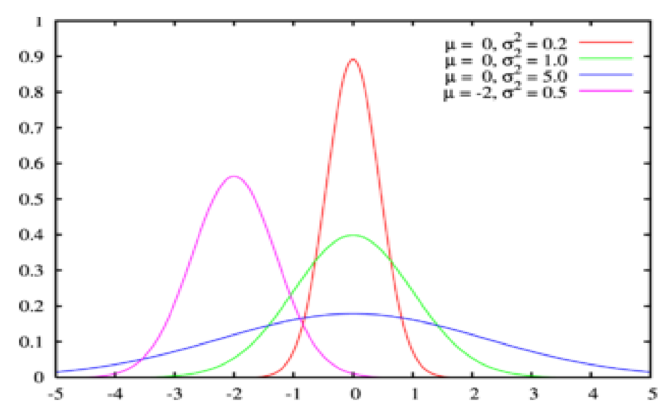
This can be viewed as a queuing system with waiting time that has service channels. Since the flows in this case are of high density, according to previous research, we can assume the time intervals between consecutive pedestrians to be distributed normally. Furthermore, the flow quickly enters a stationary state.

By service time we will refer to the time it takes for one person to pass through the bottleneck (doorway). In the described situation, this time can also be considered normally distributed.

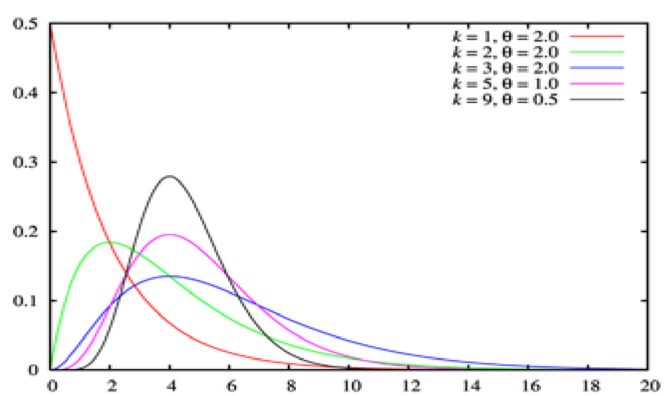
The normal law is well approximated by the Erlang law at k≥5 (Figure 3).

Thus, we have ***m*** service channels and the queue can be considered limited, containing no more than ***n*** people. The flow of requests follows a ***k***th-order Erlang distribution and service time follows an ***l***th-order Erlang distribution (Figure 3). In this way, we have a queuing system of the form E_k/E_l/m/n.

Fig 3. Flow of requests in an Erlang distribution of order k and service time.



A) Normal distribution (Gaussian distribution)



B) Gamma distribution (Erlang distribution).

Source: own elaboration.

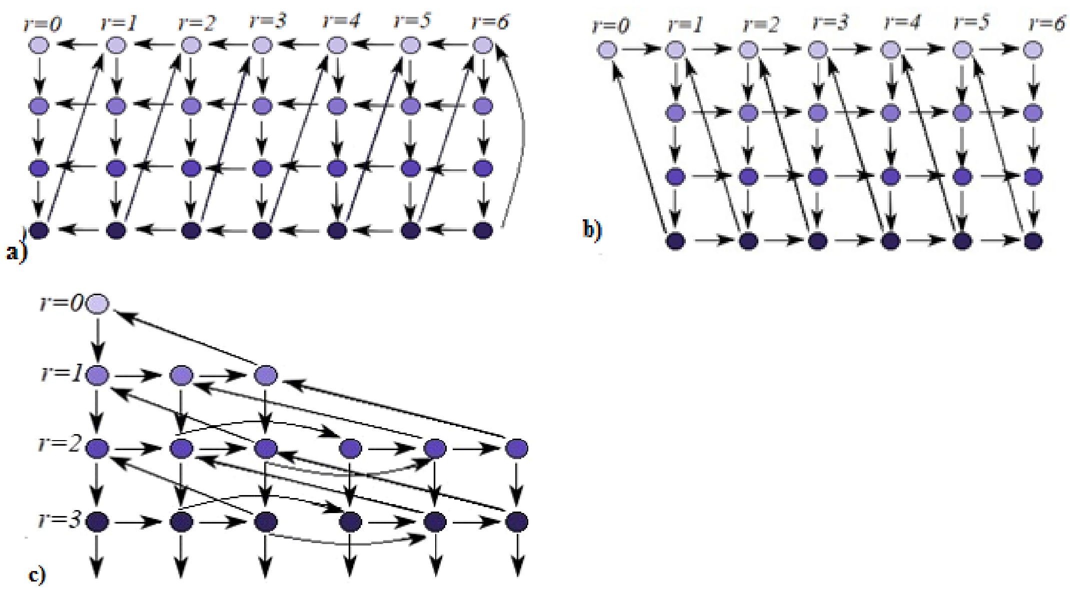
In this case, using the method of pseudostates, we can reduce the system to a Markov process.
Erlang law of the *k*th order can be represented as a sum of *k* identical exponential distributions (stages, phases or pseudostates).

For the convenience of compiling the process intensity matrices, we introduce the following numeration of pseudostates:

1) for the incoming flow (*k*th-order Erlang law E_k with intensity λ), pseudostate 1 corresponds to the receipt of the previous request and pseudostate ***k*** — to the receipt of the current request;

2) for the service process (*l*th-order Erlang law E_l with intensity μ), pseudostate ***l*** corresponds to the receipt of a service request and pseudostate 1 — to the end of service. An example of state graphs for QS E_3/M/1/n , QS M/E_l/1/n, and M/E_l/2/n is provided in Figure 4

Fig 4. Example of state graphs.



Source: own elaboration.

a) QS E_3/M/1/n; b) QS M/$E_{l/1}$/n; c) M/E_l/2/n.

Mathematical objects and notations in modeling a queuing system

Let us introduce the following notations:

U_r– a set of all microstates, in which there are *r* requests in the system;

Ω={1,2,…, ω} , ω ≤∞::

, $\omega = \omega_0 + \omega_1 + \cdots + \omega_{(m+n)}$, where ω_q — the number of states at level U_q;

(i,j,r) — a pseudostate of QS, where ***i*** — the number of the incoming flow stage, ***j*** – service stage, ***r*** – the number of requests in the system;

$j=(j_0,j_1,\dots,j_l)$ – the state of devices in service, where

j_0 the number of unoccupied devices,

j_q the number of devices at service stage ***q,*** while q≤l, 0≤j_q≤m *,* and $j_0+j_1+\cdots+j_l=m$.

We will assume (F23) that $j<j'$, if $\sum_{k=0}^{l} j_k m^k < \sum_{k=0}^{l} j'_k m^k$ (F23)

To compile the matrices, the states must be ordered. Let be the number of the QS pseudostate. We will assume that:

1) if r<r/ , then ∀i,j,i/,j/ and it is true that N(i,j,r)<N(i/,j/,r/)

2) if i<i/ , then ∀ j,j/,r and it is true that N(i,j,r)<N(i/,j/,r)

3) if j<j/ , then ∀ i,r and it is true that N(i,j,r)<N(i,j/,r)

Let us present a set Ω in the form Ω=Ω0 ∪ Ω1 ∪ … ∪ Ωm+n, while Ω_i≠Ω_j at i≠j . Furthermore, if (F24) $i < j, \quad \alpha \in \Omega_i, \vdots \beta \in \Omega_j$, then (F24)

Let $P_{Nl,Nm}(t)$ denote the probability of the system transitioning from state Nl into state Nm over the time period t.

The distribution can be considered stationary (F25), then $p_{N_m} = \lim_{t\to\infty}\left(p_{N_l,N_m}(t)\right)$, где $N_l, N_m \in \Omega$ and $\sum_{N_m \in \Omega} p_{N_m} = 1$ (F25)

Then, let $\pi(q) = \left(p_{N1}, p_{N2}, \ldots, p_{N\omega q}\right)$ denote the vector of stationary probabilities of the subset Uq.

P(t) – probability matrix of one-step transitions.

Q – matrix consisting of the intensities of transition from state N_l into state Nm.

The elements of matrix Q are as follows (F26, F27, F28):

$$\nu_{N_l,N_m} = \lim_{\tau\to\infty}\frac{p_{N_l,N_m}(\tau)}{\tau}, \quad N_l \neq N_m \quad \text{(F26)}$$

$$\nu_{N_m,N_m} = \lim_{\tau\to\infty}\frac{p_{N_m,N_m}(\tau)-1}{\tau} \quad \text{(F27)}$$

In this case, it is true that

$$\nu_{N_m,N_m} = -\sum_{\substack{N_l \neq N_m \\ N_m \in \Omega}} \upsilon_{N_l,N_m}, \quad N_l \in \Omega \quad \text{(F28)}$$

To write the equations of QS states more conveniently, we will divide matrix Q into the following blocks:

$Q_{\alpha\beta}$ – square matrix $\omega_\alpha \times \omega_\beta$, made up by the intensities of the transition from state $N_l \in \Omega_\alpha$ into state $N_m \in \Omega_\beta$.

With the chosen numbering, the elements above the main diagonal in Qαβ matrices depend only on incoming flow parameters, while those below the main diagonal depend on service characteristics.

The equilibrium equations for the system are as follows (F29):

$$\begin{cases} \pi(0)\cdot Q_{00} + \pi(1)\cdot Q_{10} = 0 \\ \pi(i-1)\cdot Q_{i-1,i} + \pi(i)\cdot Q_{i,i} + \pi(i+1)\cdot Q_{i+1,i} = 0, \quad 1 \le i \le n+m \\ \pi(n+m-1)\cdot Q_{n+m-1,n+m} + \pi(n+m)\cdot Q_{n+m,n+m} = 0 \end{cases} \quad \text{(F29)}$$

Method for constructing $Q_{\alpha\beta}$ matrix elements 1.

The elements of blocks $Q_{(r,r+1)}$ correspond to transitions from pseudostate (i,j,r) of level Ur into pseudostate (i/,j,r+1) of level Ur+1. All of these elements are non-zero except for elements at i=1, i/=k (for the Ek incoming flow), which equal kλ.

2. The elements of blocks $Q_{r,r-1}$ correspond to transitions from pseudostate (*i,j,r*) of level U_r into pseudostate ($i,j^/,r-1$) of level U_{r-1}. The elements of the block (in E_l service) will be non-zero if the following conditions are met (F30):

$$j_1 - j_1^/ = 1, \quad i_q = i_q^/, \quad 2 \le q \le l-1 \quad , \quad j_0^/ = \begin{cases} j_0 + 1, & r \le m \\ j_0, & r > m \end{cases} \quad \text{(F30)}$$

$$j_l^/ = \begin{cases} j_l, & r \le m \\ j_l + 1, & r > m \end{cases}$$

These elements then equal j_1/μ.

3. The $Q_{r,r}$ blocks are square. The elements ***below*** the main diagonal will be non-zero only if for *j* and $j^/$, there is a single ***q*** index, ***o≤q≤l***, such that j_q-j_q^/>0 , and for the remaining indices j_q-$j_q^/$≤0 . The respective elements equal j_q/μ. The rest of the elements below the main diagonal are equal to zero. These elements characterize the transition from (*i,j,r*) to ($i,j^/,r$).

The elements above the main diagonal will be non-zero only if $i^/$=i+1, in which case they will equal (kλ). The rest of the elements above the main diagonal are equal to zero. These elements characterize the transition from (i,j,r) to (i/,j,r).

Algorithm for calculating the stationary probabilities of the QS :

1) define the set of QS states $\Omega = \{(i, j, r)\}$ and its levels U_r;

2) define $Q_{\alpha\beta}$ blocks that make up the ***Q*** matrix of transition intensity;

3) find the solution of the system (20) using recurrence relations:

3.1) find the matrices (F31) of $D_i, \quad 1 \le i \le n$:

$$D_{i+1} = -Q_{i,i+1} \cdot \left(Q_{i+1,i+1} + D_{i+2} Q_{i+2,i+1}\right)^{-1}, \quad D_{n+1} = 0 \quad \text{(F31)}$$

3.2) find $\pi(0)$ from the system of equations (F32):

$$\begin{cases} \pi(0) \cdot \left(Q_{0,0} + D_1 Q_{1,0}\right) = 0 \\ \pi(0) \cdot \left(E + \sum_{k=1}^{n} \prod_{j=1}^{k} D_j\right) E_{n\times1} = 1 \end{cases} \quad \text{(F32)}$$

where ***E*** – the identity matrix, E_{nx1} – the column of units

3.3) find $\pi(i), \quad 1 \le i \le n$ using the recurrence relation: $\pi(i+1) = \pi(i) D_{i+1}$.

Calculating pedestrian flow characteristics at the exit from the event area

After determining stationary probabilities $\pi(q) = \left(p_{N1}, p_{N2}, ..., p_{N\omega q}\right)$ at the level of we can define the characteristics

of the system. Let us denote the sum of stationary probabilities at the level of as $\pi(q)_\Sigma$, then (F33, F34, F35):

1) the average number of queue requests:

$$\overline{N}_Q = \sum_{r=1}^{n}\left(r \cdot \pi(r+m)_{\Sigma}\right) \quad \text{(F33)}$$

2) the average number of requests in the system:

$$\overline{N} = \sum_{r=1}^{n}\left(r \cdot \pi(r)_{\Sigma}\right) \quad \text{(F34)}$$

3) the average waiting time in the queue:

$$\overline{T} = \frac{\overline{N}_Q}{k\lambda} = \frac{\sum_{r=1}^{n}\left(r \cdot \pi(r+m)_{\Sigma}\right)}{k\lambda} \quad \text{(F35)}$$

CONCLUSIONS

Analysis of the experience of organizing large-scale mass events points to the great importance of proper visitor flow management. To effectively utilize the checkpointing system at the entrance to the event area, it is necessary to match system parameters with the characteristics of the pedestrian flow. The study examined a method to achieve this, which relies on queuing theory. Due to the large dimensionality of the obtained system of differential equations necessary to find the probabilities of the system states, only a numerical solution is possible. To this end, we presented two solution methods.

The flow of visitors at the exit from the event area has slightly different characteristics and is discussed in the article separately. A distinctive feature is the high density of the flow. In this study, this process is described using a multi-channel queuing system with an Erlang flow of requests and Erlang service time.

The methods proposed in this paper can be referred to as mesoscopic modeling. As a result, the study provides a method for calculating the characteristics of the quality of visitor service, which can be applied in automated regulation of visitor flows using digital information boards.

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