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SOME GENERALIZATIONS AND CLARIFICATIONS ABOUT THE USE OF THE INTERVAL METHOD

ALGUNAS GENERALIZACIONES Y ACLARACIONES SOBRE EL USO DEL MÉTODO DE INTERVALO

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ABSTRACT

In the academic literature on the interval method, we have observed that the scope of an isolated investigation can hardly cover the entire extent of the knowledge generated in this field. There are a large number of problems addressed with this mathematical approach, and new applications and lines of research based on interval arithmetic periodically emerge. Given the wide range of applications, researchers often encounter significant difficulties when exploring new problems. However, we believe that instead of starting from scratch, it is very useful to synthesize some generalizations and abstractions from previous work at strategic moments. In this way, the resolution of a new problem can begin based on existing advances. Taking this as a reference, the objective of this research is to discuss some generalizations and clarifications about the use of the interval method, for which specific examples of great interest were analyzed.

Keywords: Equations, inequality, module, generalization, interval method.

RESUMEN

En la literatura académica sobre el método de intervalos se observa que el alcance de una investigación aislada difícilmente pueda abarcar toda la extensión del conocimiento generado en este campo. Existe una gran cantidad de problemas abordados con este enfoque matemático, y periódicamente surgen nuevas aplicaciones y líneas de investigación basadas en la aritmética de intervalos. Dado el amplio rango de aplicaciones, con frecuencia los investigadores se topan con dificultades significativas cuando exploran nuevos problemas. Sin embargo, se cree que, en lugar de comenzar desde cero, resulta muy útil sintetizar algunas generalizaciones y abstracciones a partir de trabajos previos en momentos estratégicos. De esta manera, se puede iniciar la resolución de un problema nuevo tomando como base los avances existentes. Tomando esto como referencia, el objetivo de esta investigación es discutir algunas generalizaciones y aclaraciones sobre el uso del método de intervalo, para lo que se analizaron ejemplos específicos de gran interés.

Palabras clave: Ecuaciones, desigualdad, módulo, generalización, método de intervalo.

INTRODUCTION

Interval arithmetic is a mathematical approach to manage uncertainty and error propagation in numerical computations. Rather than using singular values, variables are denoted as ranges between specified limits (intervals). Interval arithmetic diverges from traditional numerical analysis by accounting for interdependencies between variables and their associated uncertainties. Although conventional methods may not directly convert into interval-based algorithms, adapted approaches like rounded interval arithmetic enable practical implementations. When functions are applied to these input intervals, the resulting interval encapsulates all feasible output values across the entire domain of the input interval. Then, a major benefit of interval arithmetic is its capacity to provide assurances regarding the reliability of computed results. By encompassing all prospective values, the final interval is guaranteed to contain the true solution, despite being non-singular. This proves valuable in situations involving measurement imprecision, tolerance specifications, or inherent data uncertainty (Akkaş et al., 2004; Ganesan & Veeramani, 2005; R. Moore & Lodwick, 2003; Piegat & Landowski, 2017; Tang & Fu, 2017).

Interval analysis finds application across diverse fields, showcasing its versatility and utility. Some notable applications include mathematical programming and operator equations, where it serves to establish secure initial regions for iterative methods and provides computable adequate conditions for both existence and convergence (Tangaramvong et al., 2016; Wu et al., 2015). Within the financial domain, interval analysis has been utilized, particularly concerning internal rates-of-return (Fu et al., 2007; Sewastjanow & Dymowa, 2008). In computer analysis, interval analysis has been often coupled with automatic differentiation but also it aids in drawing function contours, ray tracing implicit surfaces, and ray tracing parametric surfaces (de Almeida Ayres & de Figueiredo, 2020; Fang et al., 2018; Sigalotti et al., 2021). In statistical problems, interval analysis techniques are applied, enabling the use of guaranteed numerical methods that furnish inner and outer approximations of sets of interest. These methodologies facilitate tasks like discovering all solutions to nonlinear equations and inequalities or identifying a global optimizer for potential multi-modal criteria. These diverse applications collectively underscore the broad scope and practical value of interval analysis across various disciplines (Moore, 1979).

Utilizing interval analysis can present a range of challenges due to various factors that come into play, such as complex problem formulations, computational hurdles, and issues with interpretation. For example, one notable difficulty is the complexity inherent in multi-attribute decision-making problems. The sheer number of variables and their potential interactions can result in situations of Pareto-inefficiency, where achieving an optimal outcome for one attribute may come at the expense of another. This complexity adds layers of intricacy to the analysis process, demanding careful consideration of trade-offs and compromises. Furthermore, uncertainties in real-world scenarios pose significant challenges when employing interval analysis. These uncertainties make it challenging to precisely define the intervals for parameters and variables. Since interval analysis relies on these intervals to compute results, the accuracy of the analysis is directly affected by the precision of these interval bounds. Dealing with uncertainties requires sophisticated methods for handling and incorporating probabilistic or fuzzy data into the analysis framework, adding another layer of complexity to the process. In addition, from the computational point of view, interval analysis can be demanding, especially when working with large datasets or highly complex models. The calculations involved in interval computations can become computationally intensive, leading to longer processing times and resource requirements. This aspect becomes particularly challenging when aiming for real-time or near-real-time analyses, where the speed of computation is crucial (Dahooie et al., 2018; Ni et al., 2022; Xu et al., 2015).

However, despite these difficulties, the interval method is crucial, for example, for solving inequalities due to its ability to provide guarantees about the solutions. By considering the intervals that contain all possible solutions to an inequality, this method ensures that any value within that interval satisfies the inequality, providing certainty in the results. This is especially valuable in contexts where inequalities represent constraints in optimization problems, allowing for the easy identification of valid ranges for the involved variables. Additionally, the interval method is essential for addressing inequalities with uncertain parameters or variables with approximate values. By working with intervals instead of single values, solution intervals can be obtained that capture all possible variations in the variable values. This results in robust solutions that take into account the uncertainty in the data, which is crucial in applications such as financial planning, where future projections may be subject to fluctuations and errors. Then, considering the huge applicability of this method. the objective of this research is to discuss some generalizations and clarifications about the use of the interval method, for which specific examples of great interest were analyzed.

DEVELOPMENT

The method of intervals has long been recognized as a natural approach for solving certain types of equations and inequalities in the school mathematics curriculum. Initially, this method was employed to solve rational inequalities of the form P(x)/Q(x) ΔO, where P(x) and Q(x) are polynomials, and Δ represents an inequality sign. The method of intervals is regarded as effective because similar inequalities were previously tackled through lengthy processes, ultimately leading to numerous equivalent systems of inequalities. Another topic within school mathematics that can be addressed using the interval method is problems involving absolute values. Traditionally, these problems were solved in schools using the standard method of expanding modules at intervals to demonstrate constancy under modular expressions. However, while this method is universal, it necessitates time-consuming calculations.

Recognizing the significance of applying the interval method to these problems, various works have emerged over time aimed at clarifying and expanding the scope of this unique method's application. However, we have also observed, in our considerations, some methodological shortcomings in certain aspects of these works. Then, in this paper, drawing from specific examples discussed in these works, we endeavor to offer valuable insights and generalizations concerning the application of the interval method. Timely and insightful comments on these generalizations serve to bring our students closer to the profound analysis of Olympiad problems, a pursuit greatly needed in our secondary school curriculum at present.

Trial points - are they necessary?

When solving rational inequalities of the form P(x)/Q(x) ΔO according to the classical version of the interval method, the first step involves marking the zeros of all factors of the form k-x+b on the number line that constitute the polynomials P(x) and Q(x) . Subsequently, the number line is segmented into intervals, and the task is to identify those intervals that satisfy the given inequality. To achieve this, a specific line is drawn to delineate these intervals. This process raises two natural questions (Abasov, 1998): How can this line be drawn to maximize its effectiveness? How can one determine the intervals where the given expression takes on positive or negative values?

Note that the line is defined in different ways. This cannot be said to be done effectively; moreover, dividing the intervals according to a constancy sign is done using trial points. We believe that at the end of the solution, selecting the necessary intervals using trial points is effective. In other words, it is possible to do without them by utilizing the properties of functions in the form of kx+b and ax^2+bx+c . Let's analyze an example?

Example 1. Solve the inequa lity

$$\frac{(2x-7)^5(-3x+2)(3-4x)^3(3-x)^2}{(-2x-3)^4(5-x)(-x)^6} \geq 0$$

Solution: In the numerator and denominator, we see several factors of the form (kx+b)r, where is a natural number.

Step I. Considering the parity of the powers of these factors, we will ensure that all expressions of the form kx+b have k>0 , if this is not already the case. We have three such factors—an odd number. This implies that after this operation, these inequalities change their signs. In the expressions $(3-x)^2$, $(-2x-3)^4 = (2x+3)^4$ and $(-x)^6$ this is automatically satisfied since their powers are even numbers

Step II. This inequality is transformed into a form convenient for solution $\frac{(2x-7)^5(3x-2)(4x-3)^3(x-3)^2}{(2x+3)^4(x-5)x^6} \geq 0.$

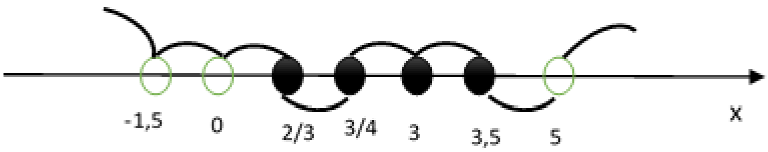
Having expressions of the form kx+b with k>0 is advantageous, as the defining line passes efficiently in this case.

Step III. Zeros of the numerator: 3.5, 2/3 , 3/4, and 3 are marked on the x-axis as black circles while zeros of the denominator: -1.5, 5, and are marked as empty circles.

Step IV. We mark these points on the number line and draw a line, starting necessarily to the right of all points and always above, since all functions kx+b>0 are greater than for (k > 0). The process until now is shown in Figure 1.

Step V. Give the answer: (-∞;-1.5)∪(-1.5;0)∪(0,2/3]∪[3/4;3.5]∪(5;∞)

Fig1. Graphical representation of the solution of example 1.



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It is importany to note that the line intersects the coordinate axis at certain points but not at others. This depends on the parity of the degree of the expression . The portion of the line above the horizontal axis corresponds to the positivity of this expression, and vice versa. Other methods of dividing directly into segments based on the sign of constancy present additional challenges for students. As demonstrated, in this solution, no trial points are necessary. But also, in even more complex problems, trial points will not be required if we utilize the properties of the functions found in the given problems more effectively.

Interval method applied to various problems with a module.

As mentioned earlier, some types of modulus problems are also addressed using the interval method, which typically involves extensive calculations. To streamline these computations, certain works suggest an alternative approach based on the concept of distance, that is, the modulus (absolute value) of a real number a, denoted by |a| - representing a distance (Enifanova, 2003; Ivanova, 2001; Kankaeva, 2004; Sevryukov, 2014). An examination of relevant literature reveals that this approach also carries some drawbacks, as it often limits the use of all functional properties of the equations (or inequalities) involved. In this paper, a technique is proposed for the efficient application of the interval method to specific types of equations and inequalities with a modulus, based on the algebraic definition of the modulus of a real number a. We will require a function as depicted in Equation 1 where are real numbers.

$$y = |x - a_1| + |x - a_2| + \cdots + |x - a_n| \qquad (1)$$

For the smallest value of this function, the formula was found in Strashevich and Brovkin (1978) and it is shown in Equation 2.

$$y_{min} = \begin{cases} (a_{2m} - a_1) + (a_{2m-1} - a_2) + \cdots + (a_{m+1} - a_m)\ at\ n = 2m \\ (a_{2m+1} - a_1) + (a_{2m} - a_2) + \cdots + (a_{m+2} - a_m)\ at\ n = 2m+1 \end{cases} \qquad (2)$$

The proof of this formula in Strashevich and Brovkin (1978) is based on the algebraic definition of the modulus of a real number and, in our opinion, looks irrational. The second proof of Equation 2 was proposed by us in when n=2m (Abasov, 2005). This approach is based on the properties of the linear function , in our opinion, it looks more effective. Since, it was by this method that we plotted the graph of function (1) separately in the cases n=2m and n=2m+1 and . And after that we made the following conclusions about solving the equation |x-a_1 |+|x-a_2 |+...+|x-a_n |=a :

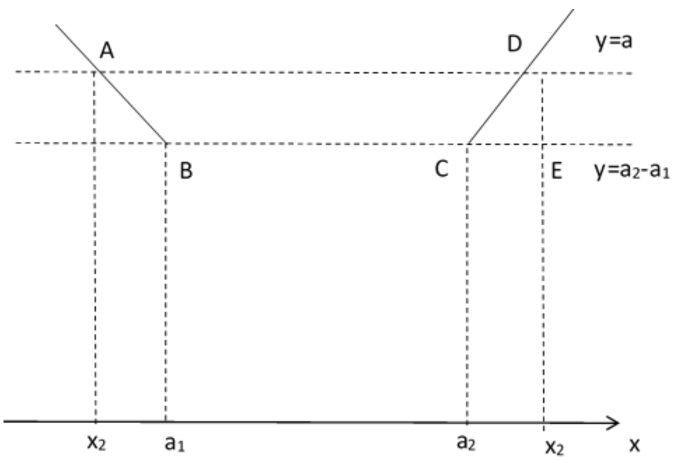
1. Equation 1 has no roots if if $a<y_{min}$..
2. In case $a<y_{min}$ any number $x \in [a_m: a_{m+1}]$ is the root of Equation 1 if n=2m, and in the case of n=2m+1 the same equation has one single root $x=a_m+1$
3. For $a>y_{min}$, Equation 1 has two roots in both cases n∈N [see (Abasov, 2005)]
4. There are no other cases for solving Equation 1.

However, how can we determine the roots (two roots) of Equation 1 in the scenario where ? The derivation of these results, along with similar questions, is not linked to the classical version of the interval method in this context. Instead, we solely rely on the respective graphs of function (1) from (2). Therefore, a particular case of Equation (1) takes the form of Equation 3.

$$\|x - a_1| + |x - a_2| = a: \quad a_1 < a_2 \qquad (3)$$

Let's explore the solution to this equation. Here n=2 , therefore the function $y=|x-a_1|+|x-a_1|$ has $y_{min}=x:x\in[a_1;a_2]$. The graph of this function is shown in Figure 2 as CD and BA break down, respectively, equal to k=2 and k=-2. Therefore, the roots of equation (5) x_1 and x_2 are located symmetrically relative to the segment $[a_1;a_2]$.

Fig 2. Graphical representation of the example 2.



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From Figure 2 it can be proofed that $DE=a-y_{min}=a-(a_2-a_1)$, $CE=DE/2=(a-y_{min})/2$. Let us denote $h=(a-y_{min})/2$ and find the roots of Equation 3 as $x_1=a_1-h$ and $x_2=a_2+h$. From Figure 2 it is clear that the proposed method allows one to find solutions to the corresponding inequalities as soon as the roots x_1, x_2 of Equation 3 are found.

This analysis may be extended for more complex expressions, more precisely the inequalities of Equation 4 and Equation 5. And for these equations (and the corresponding inequalities) similar questions can be studied. Here we recommend relying on the graphs of the corresponding functions for specific values of the parameters in this equation.

$$|x - a_1| + |x - a_2| + |x - a_3| = a: \quad a_1 < a_2 < a_3 \qquad (4)$$

$$|x - a_1| + |x - a_2| + |x - a_3| + |x - a_4| = a: \quad a_1 < a_2 < a_3 < a_4 \qquad (5)$$

As observed, in solving these problems, we opt not to employ the classical standard version of the interval method. We do not search for the roots of the equation or solutions to the corresponding inequalities separately within intervals such as $J\_1=(-\infty,a_1]$, $J2=(a_1,a_2]$, $Jn=(a_n-1,a\_n]$, $J_{n+1}=(a_n;\infty)$ repeating the same operation (n+1) times. In our view, the aforementioned method brings students closer to analysis than the classical approach. However, this does not imply that we completely abandon the interval method, as there exist numerous types of equations and inequalities involving modulus where the interval method can still be applied. Let's now examine other examples pertaining to Equation 3.

Example 2. Solve equation |x-2|+|x-4|=4

Here $a_1=2$, $a_2=4\Rightarrow y_{min}=4-2=2$, Right side $a=4\Rightarrow a>y\_{min}$, i.e. the equation has two roots. Then $h=(a-y_{min)}/2=1\Rightarrow$. $x_1=a_1-h=2-1=1$; $x\_2=a_2+h=4+1=5$ and the answer is 1;5. In Sevryukov (2014), the author solves this problem in the usual way based on the concept of distance, although the formulas have been known for a long time.

Example 3. Solve equation |x-1|+|x-3|=2

Here a_1=1, a_2=3, a=2⇒y_{min}=a_2-a_1=2=a⇒x∈[1;3] which is the root of this equation [see Sevryukov (2014) or Kankaeva (2004)]. In Sevryukov (2014) the author solves this problem based on the following properties of the module (without proof): if |x-a|+|x-b|=ba : (a≤b)⇒x∈[a;b] : is the root of this equation. Here a_1=a, a_2=b⇒y_{min}=ba, and everything converges on point 2 (Ivanova, 2001).

Example 4. For what values of the parameter a does the equation |x+3|+|x-1|=a have no solutions; it has one solution, two solutions, and infinite number of solutions.

Here a_1=-3, a_2=1, and the right side is . It is clear that y_{min}=1+3=4. We immediately go to Equation (3) and:

1. The equation has no roots if a<y_{min}, and a<4 .
2. The equation has countless roots if a<y_{min} , and a=4 .
3. The equation has two roots if y_{min}, and a>4.
4. The equation does not have the same root for any value of a .

Kankaeva (2004) solves this problem in three ways: 1) Algebraic method, 2) Geometric method on a straight line, 3) Geometric plane method. There is a parameter in the equation. Therefore, all three of these methods involve large calculations and subtle reasoning. In this sense, we did not consider it necessary to bring them to the end.

What have been expressed for y_{min} is correct for all x∈[a_m,a_{m+1}] In this case |x-a_1 |+|x-a_2 | can be seen even from Figure 2. And Equation 3 is only at one point x=a_m+1 (Abasov, 2005). Let's look at one example where this remark plays a significant role in effectively solving this problem.

Example 5. Solve equation |x-2|+|x-1|+|x+1|+|x+2|=6

In Sevryukov (2014) the author chooses the usual classic version of the interval method. The numbers-2,-1,1,2 are marked on the number line - the zeros of the terms and five intervals are obtained on it. At each interval, this equation is solved sequentially and the roots of this equation are obtained in five stages. So, it is arrived at the answer: [-1,1]. The response received, i.e. segment [-1,1] prompts us to refer to the formula (here n=4 is an even number). Since y_{min}=a, i.e. x∈[-1,1] is the root of this equation [see (Abasov, 2005)] we can conclude the following:

1. if the result would be y_{min}>6, then this equation has no roots.
2. if y_{min}<6 – then it has two roots. The work developed by Abasov (2005) will tell us what intervals these two roots are located on.

The equations we have considered (as well as modulus inequalities) have a special form, i.e. the left side of the equations consists only of the sum of the modules and the coefficients for all x are the number k=1. To explore similar questions for Equation 6 and Equation 7 (and corresponding inequalities) we must find y_{min} from the left sides of these equations. These values were also found in the work by Abasov (2005).

$$|k_1 x - b_1| + |k_2 x - b_2| = a \quad (6)$$

$$|k_1 x - b_1| + |k_2 x - b_2| + |k_3 x - b_3| = a \quad (7)$$

For example, the function y=|k_1 x-b_1 |+|k_2 x-b_2 | leads to y_{min} shown in Equation 8.

$$y_{min} = min(k_1, k_1)\left|\frac{b_2}{k_2} - \frac{b_1}{k_1}\right| \quad (8)$$

It is clear that the formula y_{min} =a_2-a_2 (a_1<a_2) from Equation 3 is a special case of Equation 8. Then, let's analyze one example where Equation 8 is effectively applied.

Example 6. At what values of the parameter k does the equation |x-2k|+|4x+k|=27 (11) has exactly two roots (Enifanova, 2003)?

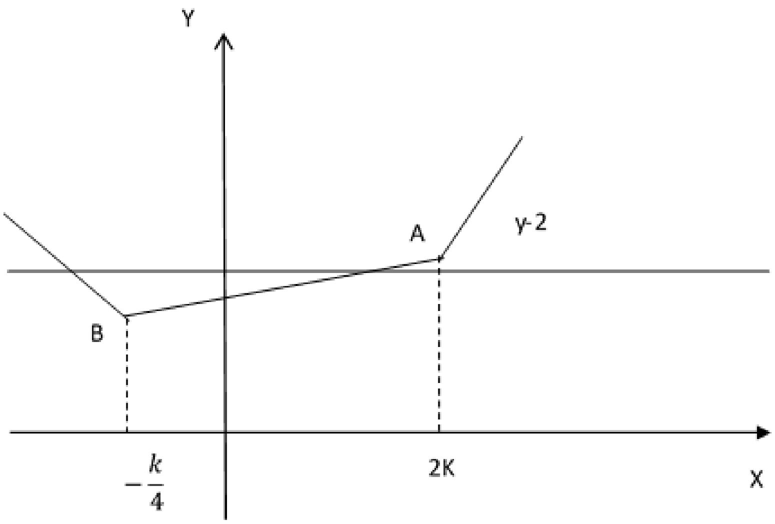
The author automatically understands that k>0, although there is no explanation for this. We will present the solution as briefly as possible.

Method I. Consider the function of Equation 9

$$y = f(x) = |x - 2k| + |4x + k| \quad (9)$$

The graph of such a function is a broken line of three links as shown in Figure 3.

Fig 3. Graphical representation of example 6.



Source: own elaboration.

Then points A and B are break points and: 1) x-2k=0, x=2k, f(2k)=9|k|, A(2K,9|k|); 2) 4x+k=0, x=-k/4, f(-k/4)=9/4|k|, B(-k/4;9/4|k|). But, where does the function has a minimum, at point A or B (?) According to Figure 3 it is clear that the equation has exactly 2 solutions if the value at the minimum point is less than f (x) = 27. This means that one of two inequalities must be satisfied: f(2k)<27 or .f(-k/4)<27. Since f(2k)=9|k|, then ⇒9|k|<27⇒-3<k<3. Therefore, f(-k/4)=9/4 |k|<27⇒|k|<12⇒-12<|k|<12. The combination of the resulting intervals will lead to the answer: (-12,12). Some interesting questions on this solution are:

1. It is clear that for k=0, Equation 9 has two roots |x|+|4x|=27⇒|x|=27/5⇒$x_{1,2}$=±27/5 . This case is not considered in the decision.
2. It is clear that the author assumes the parameter k>0 (why?), whereas how to understand the expressions f(2k)=9|k| (not 9 k) and f(-k/4)=9/4|k| (not 9/4 k) ?
3. It can be seen and from the calculation it is clear that function of Equation 9 has y_{min}=f(-k/4)=9/4|k|(-9/4 k,k>0) . Then, how to understand the two inequalities f(2k)<27 and f(-k/4)<27?
4. What is the meaning of combining two inequalities: -3<k<3 and -12<k<12 from which the answer is obtained: (-12,12) : However, the number k = 0∈(-12;12) , but this is not discussed in the decision?

To avoid these confusions, we add the following comments to this problem.

- When k=0, Equation 9 has two roots $x_{1,2}$=±27/5 ;
- Let k>0⇒y_{min}=f(-k/4)=|-9/4 k|=9/4 k<27⇒0≤k<12
- Let k<0⇒y_{min}=f(-k/4)=|-9/4 k|=-9/4 k<27⇒9/4 k>27⇒-12≤k<0⇒-12<k<12

Method II. On the plane we introduce a rectangular coordinate system and on it we construct straight lines x = 2k and x = -k/4 . These lines divide the plane into 4 parts (similar to how the number line is divided into 4 intervals). Then the task is solved separately on these parts. This technique, to some extent, can conditionally be called a generalization of the method of intervals to the case of a plane, and therefore we stop presenting this solution here. Now there is a simple solution to this problem. We saw that the graph of the functions $y=|x-a_1|+|x-a_2|$ and $y=|k_1 x-b_1|+|k_2 x-b_2|$ and fundamentally little noted. Therefore, Equations 3 and 6 will have two roots if y_{min}<a. We apply Equation 8 for the function y=|x-2k|+|4x+k|. Then $y_{\_min}$=min(k_1;k_2)|b_2/k_2 -$b_1/k_{\_1}$ |=min(1;4)|-k/4-2k/1|=9|k|/4<27⇒|k|<12⇒-12<k<12 .

Now the main question. Why does this happen? Why is a seemingly not very complicated example solved so difficult and tediously (especially method 2)? In parallel with this, Equation 8 was proved for the function y=|k_1 x-b_1 |+|k_2 x-b_2 | . If we analyze these two problems, we can find many common theoretical points in them. Some part of these moments the corresponding logic gives us a formula like Equation 8. And when we apply formulas like this to a specific problem, each time we do not start the solution from scratch, but from some basic stage. And this gives such an effect. Therefore, in large topics (such as the interval method), it is important to make as many generalizing formulas, conclusions and valuable clarifications as possible so that they can then be effectively applied to specific problems.

In addition, in Abasov (2005), a more general equation of the form is considered as shown in Equation 10.

$$\pm|k_1 x - b_1| \pm |k_2 x - b_2| \pm |k_n x - b_n| = kx + b \quad (10)$$

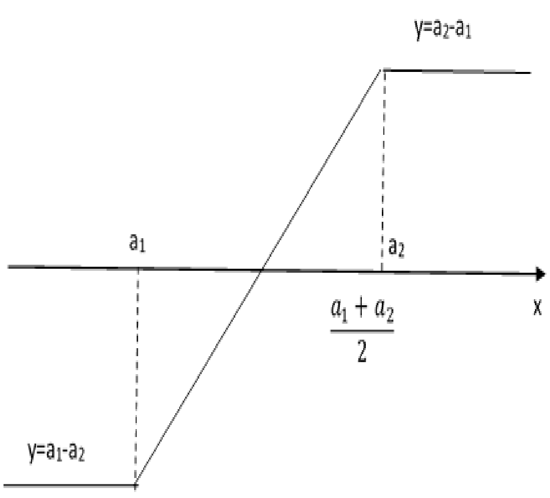
A different method is used to solve this equation as the above approaches apply here. Let's write this equation in the form F(x)=0. Having found the values of the function F(x) at the points $x_0=b_i/k_i$ (i-1,n) and additionally, at some specific points it is easy to construct a schematic graph of the functions . Using this graph, you can find the roots of Equation 10, as well as determine the solution to the inequality ***F(x)ΔO***. The work by Abasov (2005) also considers other effective variants of the interval method. In some examples, these different options are used in synthesis, which to some extent facilitates the calculations that are characteristic of the interval method. Several examples of this technique are also given in Abasov (2005).

Let's now go deeper into Equation 3 which despite of its simple form, some useful and interesting generalizations can be made. For example, consider the function of Equation 11.

$$y = |x - a_1| - |x - a_2| \ (a_1 < a_2) \quad (11)$$

It is easy to prove that this function has and the graph is as shown in Figure 4.

Fig 4. Graphical representation of problem of Equation 11.



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The solution to Equation 11 is determined as shown in Equation 12 [see (Abasov, 2005)].

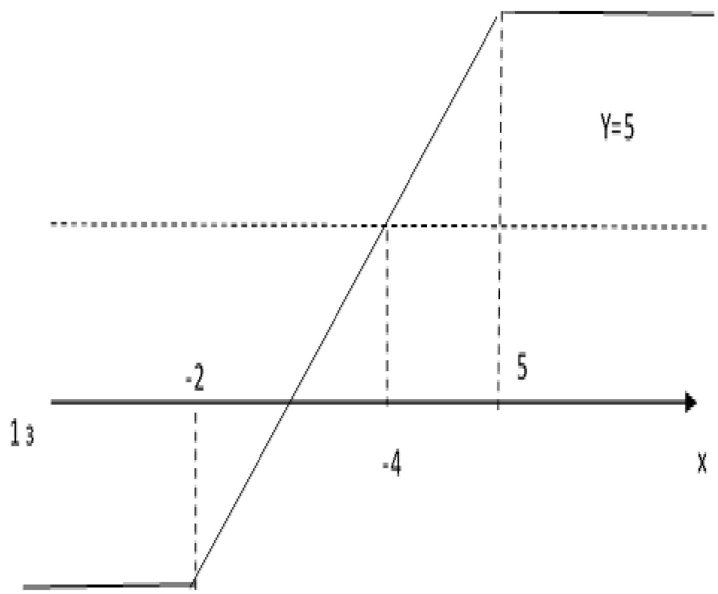
$$x = \begin{cases} \forall x \in (-\infty; a_1)\ with\ a = a_1 - a_2 \\ \dfrac{a + a_1 + a_2}{2} \in (a_1; a_2)\ with\ a \in (a_1 - a_2; a_2 - a_1) \\ \forall x \in (a_2; \infty)\ with\ a = a_1 - a_2 \end{cases} \qquad (12)$$

From Figure 4, then it is easy to find a solution to the inequality of Equation 13 where we first find the root of Equation 3.

Example 7. Solve the inequality |x+2|-|x-5|>5 (Ivanova, 2001).

Considering that the absolute value is a distance, a standard, long calculation using the interval method is used to get the answer (4,∞) The straight line y=5 intersects the graph of the function y=|x+2|-|x-5| at one point. So we can find the point $x=(a+a_1+a_2)/2=(5+(-2)+5)/2=4$ (Figure 5). From Figure 5 it is clear that the inequality |x+2|-|x-5|<5 has a solution (-∞,4). As can be seen, knowing the graph as explained in Example 6, we can easily the problem for any parameters a_1, a_2, a.

Fig 5. Graphical representation of Example 7.



Source: own elaboration.

CONCLUSIONS

Interval arithmetic is of great importance in mathematics because it allows us to represent and operate with ranges of values instead of specific numbers. This is highly useful when working with quantities that inherently possess some uncertainty or variability. Intervals provide an idea of the range within which the real value could lie and how this range propagates through operations. In particular, interval arithmetic is highly applicable to solving inequalities and equations involving absolute values or modulus. These mathematical expressions represent constraints but allow for some variability in the values that satisfy the constraint. By using intervals, one can rigorously determine the ranges of values that fulfill the inequalities or equations. This enables the establishment of precise bounds within which any valid solution must lie. Specifically, this capability is invaluable in many applications of mathematics such as optimization, numerical analysis, statistics, and more. In this research, based on specific examples we strived to offer valuable insights and generalizations about the application of the interval method. We believe that the comments included on these generalizations will serve to further disseminate and facilitate the use of the method.

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